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SURFACE WATER WAVE THEORIES

By Martin A. Mason, M. ASCE

HYDRAULICS DIVISION

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PAPERS

SURFACE WATER WAVE THEORIES

By Martin A. Mason,[1] M. ASCE

Synopsis

The formation and action of surface water waves is a subject of vital concern to a large proportion of the engineering profession, and yet little is actually known regarding these phenomena. This paper discusses the characteristics of oscillatory surface waves and summarizes the development of pertinent theory. The more important equations that characterize wave formation and movement are presented.

The method by which waves are believed to be generated is described, and a theory of the growth of waves is formulated. Several charts provide convenient means of determining wave characteristics and wave effects from a knowledge of the limiting factors. Other subjects, including wave refraction, diffraction, and reflection, are also briefly treated.

Several formulas are available for determining the action of waves on structures. Although these formulas are not perfect they offer convenient design criteria when used intelligently and carefully.

The paper reviews available knowledge on transportation of beach and bottom material and makes recommendations for likely fields of future research. A brief summary of the use of models in the study of wave problems is also included.

Introduction

The theories and discussions presented in this paper are based on both hypothesis and analysis of observed physical phenomena. They should be considered as representing the best concepts of the present (1950) subject to modification, or even major revision, as more is learned about a complex natural phenomenon that has not yet been completely defined.

Note.—Written comments are invited for publication; the last discussion should be submitted by September 1, 1952.

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The waves to be considered are the surface waves usually observed on large bodies of water. They are periodic disturbances of the surface layers of the water, generated by wind and moving under the control of gravity and inertia. They induce a steady state of oscillation in the water over whose surface they move and are of such height and period as to break, forming surf, on a sloping shore. They occur on oceans, lakes, reservoirs, rivers, or any body of water in which they can be formed by the action of wind blowing over the water surface. They are not tidal waves, surge waves, ripples, or waves of translation.

The water surface pattern at any time may be the result of a single series of waves of approximately uniform character (rarely) or (usually) of one or more series of waves of varying character. In general, the most obvious characteristic of the water surface under wave action is confusion. In contrast, the waves to be discussed in this paper are assumed to be orderly arrays of uniform character, each wave following another in mathematical precision.

It will be apparent to the most casual reader that the waves to be discussed probably never occur in nature as they will be described. Not only do uniform individual waves of unlimited lateral extent probably not occur but neither do single trains or series of waves of uniform character and appearance occur. This very complexity has forced students of oscillatory wave phenomena to attempt simplification of the problem, leading obviously to the basic hypothesis that any wave situation is the result of the simultaneous existence of one or more series of waves whose character may be considered to be uniform over short intervals of time but variable over long intervals. In this simplified situation the various component wave systems each can be studied as a series, or train, of waves of uniform characteristics.

No methods are known by which the components of a natural wave situation may be separated or assumed uniform wave systems combined to simulate natural conditions. Nevertheless, it is believed that an arbitrary uniform wave system can be defined such that its effects, insofar as engineering problems are concerned, approximate closely those of the natural wave system. This arbitrary system is considered to be composed of waves having a height equal to the average height of the one-third highest natural waves present and a period equal to the average period of the most prominently defined waves present.

This stratagem has produced satisfactory results in the solution of several engineering problems, but care must be exercised in any specific case to insure its admissibility as an adequate method of analysis.

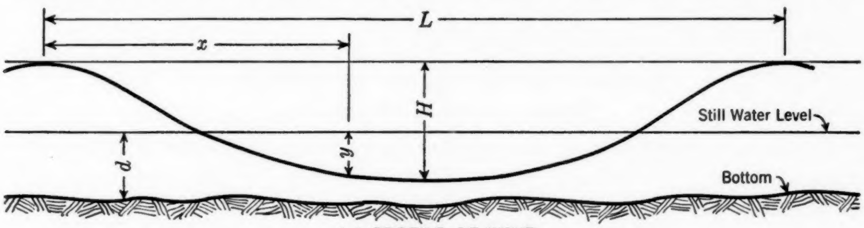
The reader will be warned by these preliminary words that no formalized solutions of engineering problems involving waves should be expected and that the uninformed and indiscriminate use of the information that follows may lead to wholly erroneous solutions.

Characteristics of Oscillatory Waves

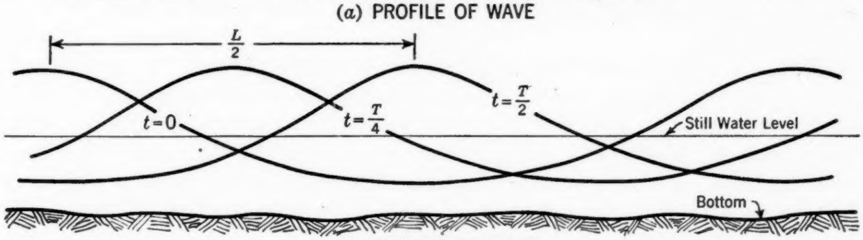
Definitions of Wave Characteristics.—Oscillatory waves can be defined by certain measurable characteristics (Fig. 1). The length of the wave, L, is the linear distance in the direction of wave travel between any two identical elements of the wave surface, for instance, crest to crest, or trough to trough. The crest length of the wave, or linear distance measured along the crest of the wave,

is a measure of the extent of the wave; it is the width or lateral extent of the wave. The height of the wave, H, is the vertical difference in elevation between the highest and lowest parts of the wave surface, or, more simply, the vertical distance from trough to crest. The period of the wave, T, is the time interval required for passage of a single wave past a fixed point; it may be measured by a variety of obvious means.

The shape of the wave is the geometry of a single wave surface that is usually measured as an instantaneous section of the surface. No adequate definition of wave shape in the form of a shape factor has been developed. The



(a) PROFILE OF WAVE



(b) ADVANCE OF WAVE

FIG. 1.—SURFACE WAVES

steepness of the wave is defined by usage, but incorrectly, as the ratio of the length to the height of the wave, or its reciprocal. This ratio has little physical significance; but it is convenient since wave height and length are measured easily, and the ratio has been found useful in defining certain effects of waves. The velocity of the wave, C, or wave celerity, is the rate of travel of any element of the wave surface in the direction of travel of the wave. It can be measured by several methods, the most popular of which is the timing of passage of a wave crest over a known distance.

A wave group is an identifiable short series of waves. A characteristic identifying wave groups is that, although the individual waves leading or ending the group cannot be followed in their travel for more than short distances before they disappear, the series of waves of which they were a part maintains an observable identity. The phenomenon is illustrated by the pattern of bow waves caused by passage of a ship. The pattern and the wave group forming the pattern is discerned easily as the waves break on shore. A wave train is any series of waves, which may include one or more wave groups, all travelling in the same direction.

Certain other features of oscillatory waves are of importance in engineering and will be mentioned without extensive discussion. The movement of oscil-

latory waves past a fixed point entails periodic variations in the motion of the water particles, in the water pressure at any point affected by the motion, and in the surface elevations, each quantity passing through a continuous series of values and returning to its initial value at regular intervals of time equal to the wave period.

Particular note should be made of the fact that only the wave form shows continuous movement in the direction of wave travel, the water particles themselves being moved in a cyclic fashion but not transported continuously with the wave. In fact, except for a small secondary transportation effect known as mass transport, the water particles remain in their original area, although the wave form passes on to ultimate dissipation.

Development of Theory of Progressive Oscillatory Wave Motion

The study of oscillatory wave motion can be traced through scientific literature to Leonardo da Vinci and beyond. However, in this discussion it is sufficient to remark that these early students were led to their theoretical concepts by observation of some of the now more obvious characteristics of wave motion.

F. V. Gerstner is usually credited as being the first, in 1802, to formulate a general theory of oscillatory waves without, however, specifying the applicable field of the theory. This deficiency was supplied by the experimentation of E. Weber and W. Weber, who demonstrated some ingenuity by employing water, mercury, and brandy to study the effects of liquid specific weight. Although Mr. Gerstner postulated a theory of oscillatory wave motion, his theory is based on observations of the characteristics of solitary waves; therefore, the theory's applicability to the progressive oscillatory wave regimen is subject to some doubt.

Early theories did not adequately define observed wave motion and were in themselves conflicting. The state of knowledge in the early 1800's may be described by the reported pungent observation of Lord Rayleigh, who said that it might be possible to devise more elaborate mathematical theories that would not conflict with physical laws but that it would not be certain that waves so described actually occurred in nature!

The accepted basic theory of oscillatory wave motion was developed by G. B. Airy, G. G. Stokes, J. W. S. Rayleigh, T. Levi-Civita, D. J. Struik, and others. These students started from the same observed characteristics as did Mr. Gerstner but their philosophy differed. All the theories are based, in the first instance, on the observation that the water particles involved in oscillatory wave motion move up and down, as well as to and fro.

Mr. Gerstner sought to determine if the simplest form of harmonic motion, that is, the uniform movement of particles in a circular path or orbit, was consistent with the dynamic and continuity equations; thus he arrived at his so-called "trochoidal theory." Stokes criticized this theory on two points; first, the limiting shape of the crest of a Gerstner wave would be that of a razor edge of infinitely small curvature; second, the Gerstner wave contains vortices—an impossible condition under natural circumstances. Experiments and observa-

tions made in recent years support the theory developed by Stokes following his critique of Gerstner's theory. The Stokes theory, as modified by other contributors, is accepted thus far as the best theoretical solution of the problem.

The Stokes theory is based in a mathematical sense on the Euler equations of motion and the concept that there exists a velocity potential satisfying the Laplace equation for certain boundary conditions. For the purposes of this paper the elegant mathematics of the development of the theory will be neglected, and only the expressions defining the wave characteristics of interest to engineers will be stated.

The form of the wave in water deeper than one half the wave length is given by the expression

$$y = \frac{H}{2}\cos\frac{2\,\pi\,x}{L} + \frac{\pi\,H^2}{4\,L}\cos\frac{4\,\pi\,x}{L}$$

$$\frac{(e^{2\pi d/L} + e^{-2\pi d/L})\,(e^{4\pi d/L} + e^{-4\pi d/L} + 4)}{2\,(e^{2\pi d/L} - e^{-2\pi d/L})^3} \qquad (1a)$$

and in water depths less than one half the wave length by

$$y = \frac{H}{2}\cos\frac{2\,\pi\,x}{L} - \frac{\pi\,H^2}{2\,L}\cos\frac{4\,\pi\,x}{L}$$

$$\times\frac{(e^{2\pi d/L} + e^{-2\pi d/L})\,(e^{4\pi d/L} + e^{-4\pi d/L} + 4)}{(e^{2\pi d/L} - e^{-2\pi d/L})^4} + \frac{\pi^2\,H^3}{8\,L^2}\cos\frac{6\,\pi\,x}{L}$$

$$\times\frac{5\,(e^{12\pi d/L} + e^{-12\pi d/L}) + 14\,(e^{8\pi d/L} + e^{-8\pi d/L}) + 19\,(e^{4\pi d/L} + e^{-4\pi d/L}) + 32}{(e^{2\pi d/L} - e^{-2\pi d/L})^6} \qquad (1b)$$

In these expressions d is the water depth, H is the wave height, L is the wave length, y is the distance of surface above or below still water level, and x is the distance along the wave length in the direction of wave motion. Actual wave shapes agree with these theoretical shapes sufficiently well to constitute confirmation of the expressions.

The velocity of travel of the wave form (wave velocity in common terminology) in water of any depth is expressed as:

$$C = \left[\frac{g\,L}{2\,\pi}\,\frac{e^{2\pi d/L} - e^{-2\pi d/L}}{e^{2\pi d/L} + e^{-2\pi d/L}}\right]^{\frac{1}{2}} = \sqrt{\frac{g\,L}{2\,\pi}\tanh\frac{2\,\pi\,d}{L}} \qquad (2)$$

A higher degree of approximation to the theoretical true velocity is possible by the addition of terms involving the wave height, but the improvement of accuracy obtained does not justify the use of the longer form for most engineering purposes.

It should be noted that as the depth increases the value of Eq. 2 approaches $\left(\frac{g\,L}{2\,\pi}\right)^{\frac{1}{2}}$. As the depth decreases $\tanh 2\,\pi\,d/L$ approaches $2\,\pi\,d/L$ in value and Eq. 2 takes the form $C = \sqrt{g\,d}$. This is the well-known equation for velocity of long waves of small amplitude. Fig. 2 is a useful graphical representation of Eq. 2. The applicability of Eq. 2 to actual conditions has been confirmed satisfactorily by observations and experiment.

A word of explanation may not be amiss here. The velocity defined by Eq. 2 is the rate of travel of an individual wave form over the water surface. This velocity does not represent the speed of travel of the water itself; neither does it represent the speed of a series of waves except under certain circumstances. The velocity of the water is that of the water particles participating

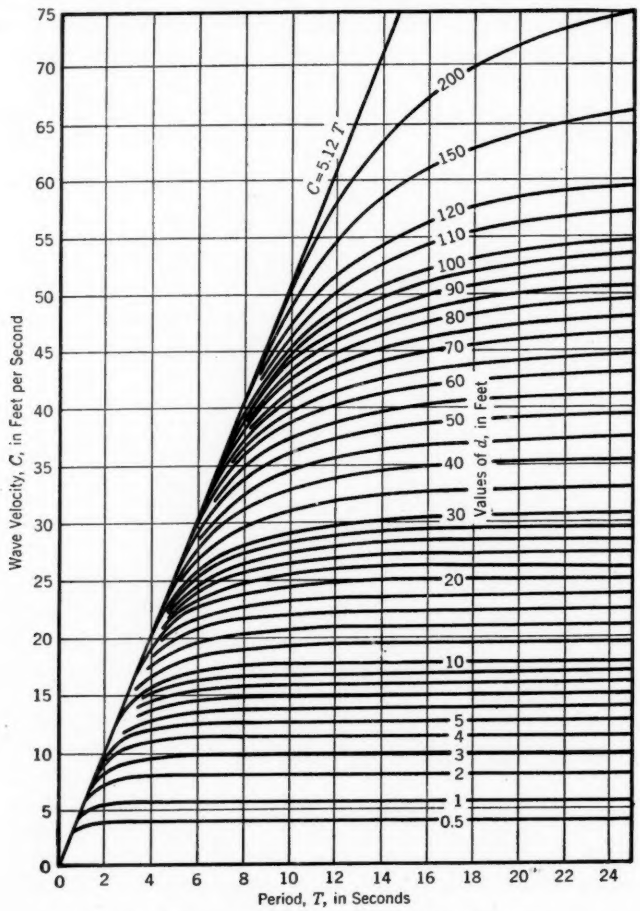


FIG. 2.—CHARACTERISTIC CURVES OF WAVE VELOCITY

in the wave motion. These particles follow orbits that vary in shape from essentially circular paths for wave motion in deep water to flat elliptical paths for wave motion in shallow water. Typical paths of water particles are shown on Fig. 3. The dashed lines in the left section of Fig. 3 are the streamlines, that is, lines everywhere parallel to the flow. The direction and length of the arrows indicate direction and velocity of orbital motion. The right section of Fig. 3 shows the particle trajectories (the paths described by individual particles of fluid).

The expressions for the horizontal and vertical components of velocity of the water particles occupying an average position at depth d_i below the surface are:

(*a*) Horizontal component:

$$u = \frac{\pi\,\alpha}{T} \sin 2\,\pi \left(\frac{x}{L} - \frac{t}{T}\right) \qquad (3a)$$

(*b*) Vertical component:

$$w = \frac{\pi\,\beta}{T} \cos 2\,\pi \left(\frac{x}{L} - \frac{t}{T}\right) \qquad (3b)$$

in which

$$\alpha = H \frac{\cosh \dfrac{2\,\pi\,(d - d_i)}{L}}{\sinh \dfrac{2\,\pi\,d}{L}}$$

and:

$$\beta = H \frac{\sinh \dfrac{2\,\pi\,(d - d_i)}{L}}{\sinh \dfrac{2\,\pi\,d}{L}}$$

α and β are the horizontal and vertical full amplitudes of motion at depth d_i; T is the period of the wave; t is some time interval; x and L have their previous significance.

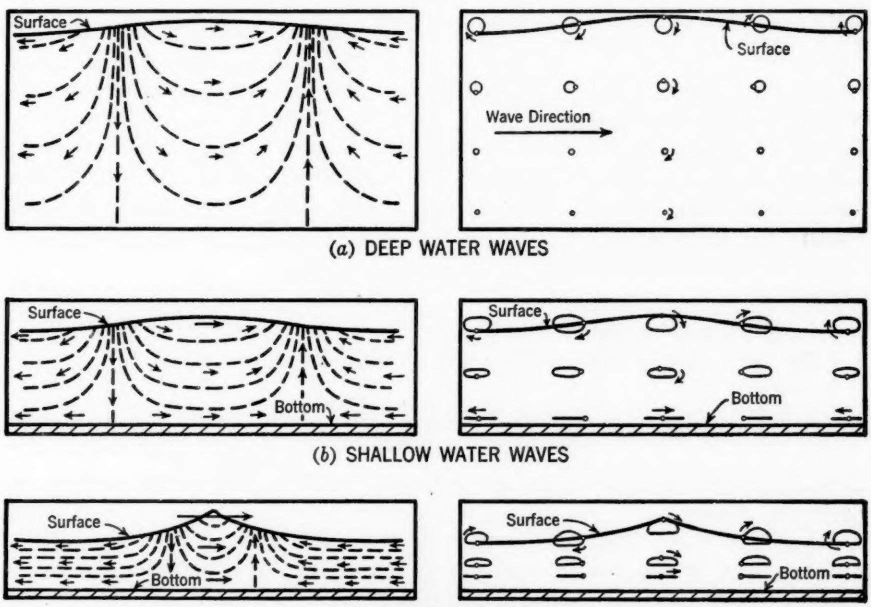


Fig. 3.—Typical Paths of Water Particles in Orbital Wave Motion

It is apparent that these are the velocities defining the dynamics of the wave, rather than the wave velocity as given by Eq. 2. An important characteristic of oscillatory surface waves is that the orbital velocities decrease with increasing depth and disappear at depths below about one half the wave length.

When waves are travelling in deep water (depths exceeding $L/2$), the water particles in the wave move approximately in circles, with the size of the circles decreasing with depth and vanishing at a depth about equal to one half the wave length. Their velocity in the orbit is not uniform, being greatest near the top of the orbit, with the result that with the completion of each cycle (wave period) the particles have advanced a short distance in the direction of progress of the wave, as shown in Fig. 4. There is thus a mass transport of water in the direction of travel of the wave (16).[2] The velocity of the transport is high for steep high waves and low for long period waves of low height. In most engineering problems mass transport may be neglected, but its existence must always be recognized in the analysis of any wave action problem.

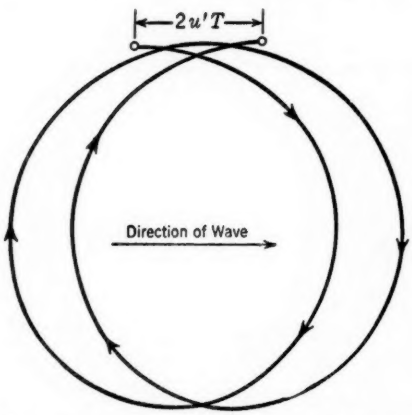


FIG. 4.—ORBITAL MOTION OF A WATER PARTICLE IN A DEEP WATER WAVE

In shallow water (depths less than $L/2$) the orbital paths are ellipses, becoming flatter with decreasing depth by reason of the reduction of the vertical motion of the water particles, and reaching a limit, before the wave breaks, of to-and-fro horizontal motion at the bottom. After the wave breaks, orbital motion no longer exists and is replaced by motion parallel to the bottom of a nature similar to variable, reversing flow. The velocity distribution of mass transport is given by

$$\bar{U} = H^2 e^{4\pi d_i/L} \left(\frac{g\,\pi^3}{2\,L^3} \right)^{\frac{1}{2}} \quad \text{.................... (4)}$$

and is shown graphically on Fig. 5.

The total volume of transport per unit width of wave crest is

$$G = H^2 \left(\frac{g\,\pi}{32\,L} \right)^{\frac{1}{2}} \quad \text{........................ (5)}$$

Experiments confirm the existence and most details of the orbital motion required by theory.

It will be noted that wave motion involves differences in elevation of the water surface, or potential energy, and motion of the water particles, or kinetic energy. Mathematical studies, confirmed in gross by experiment, lead to the conclusion that the potential and kinetic energies are equal. The energy in a

[2] Numerals in parentheses, thus: (16), refer to corresponding items in the Bibliography (see Appendix).

wave at any instant (per unit crest length, or width) is then the sum of the kinetic and potential energies of the wave. The kinetic energy is the summation of the individual kinetic energies appertaining to the orbital velocities discussed above; the potential energy is computed from the elevation or depression

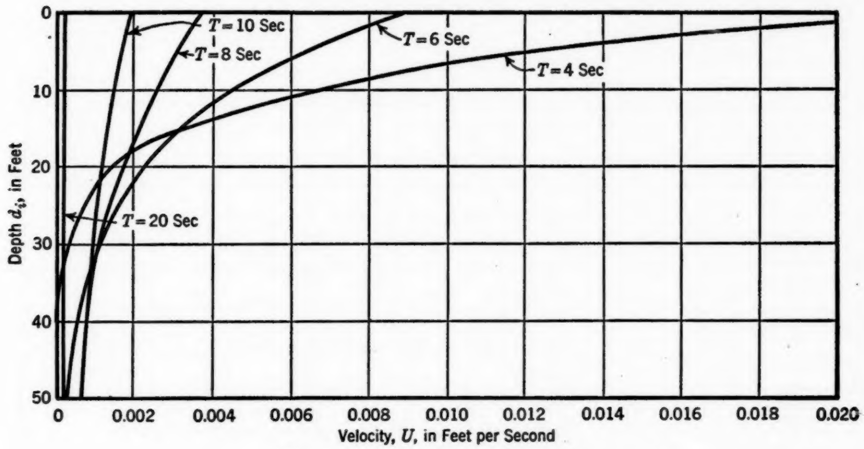


FIG. 5.—RATE OF MASS TRANSPORT

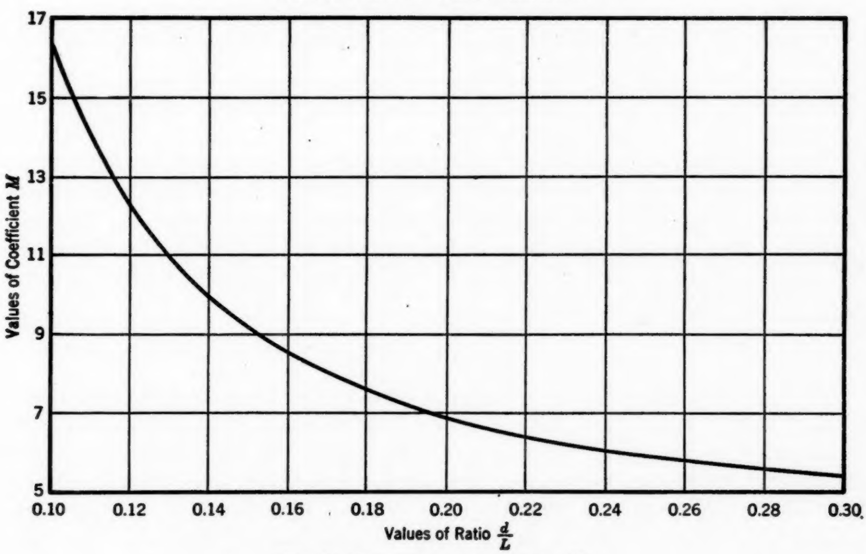


FIG. 6.—VALUES OF COEFFICIENT M

of the water surface from the still water surface. The total energy can be expressed by

$$E = \frac{w\,L\,H^2}{8}\left(1 - M\,\frac{H^2}{L^2}\right) \qquad (6)$$

in which w is the specific weight of water, and M is a number depending in value upon some function of the ratio d/L (see Fig. 6).

The total energy defined by Eq. 6 is available for the application of forces to engineering structures only as the wave is destroyed by action on the structure. For example, all the wave energy is destroyed when a wave breaks and rushes up on a shore; whereas, when a wave breaks offshore in whitecaps, only a portion of the energy is lost and the wave continues its advance.

A graph showing the energy content per foot of crest width of waves of various heights and lengths is shown as Fig. 7.

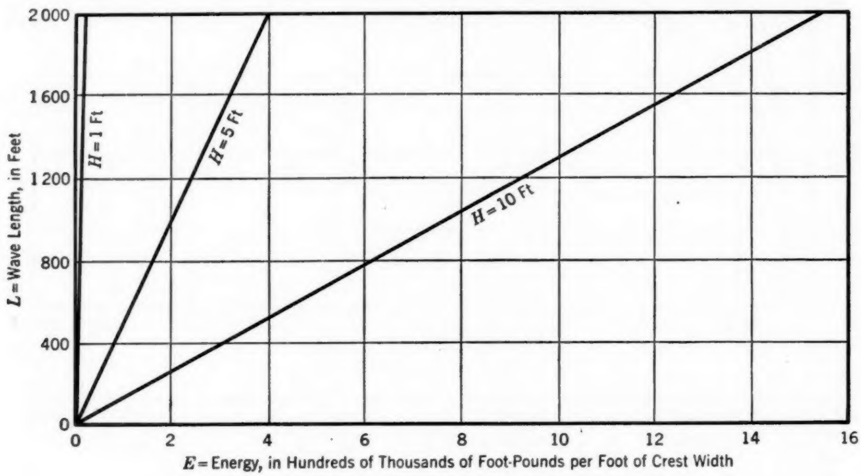


Fig. 7.—Graph of Deep Water Wave Energy

The partition, or distribution, of energy in the wave should be considered when designing works to resist wave action; yet little is known of the time rate of transformation of wave energy during destruction of wave action. Although studies of this matter have been made, no conclusive results are available. For the solution of important problems of this type, recourse to models appears to offer a satisfactory approach.

Oscillatory Wave Generation

Perhaps the greatest deficiency in wave knowledge confronting engineers concerned with wave problems is the lack of information on the wave action to be expected at any given locality and time. Very few observations of wave action in nature have been made, although a few wave observation stations providing continuous measurement over limited periods of time have been established. Data available from these stations can be obtained from the Waves Project, University of California, at Berkeley, Calif., or the Beach Erosion Board (Corps of Engineers, United States Army) at Washington, D. C. Wave measurement stations are difficult and expensive to install and maintain. Furthermore, the time schedule of many engineering works will not permit the long delay required to obtain observational data. Attempts have been made, for these and other reasons, to develop methods of predicting wave conditions from easily available synoptic weather charts, or similar data from which wind

movement over the water area concerned may be determined. These prediction methods are based on two slightly different hypotheses. Both hypotheses presume the transfer of energy from the wind to the water and some initial roughness of the water surface (see Fig. 8). The hypothesis of H. U. Sverdrup and W. H. Munk will be discussed first, in essentially the words of the originating authors.

The Sverdrup-Munk Theory.—The area in which waves are formed is called the generating area. In such an area waves receive energy from the wind by two processes: by the push (normal pressures) of the wind against the wave crests and by the pull or drag (shear) of the wind on the water.

The energy transfer by push depends upon the difference between wind velocity and wave velocity. If the waves advance with a speed much less than that of the wind, the push is great; but if the two velocities are equal, no energy is transferred. If the waves travel faster than the wind they receive no energy by push, but on the contrary they meet an air resistance comparable to the air resistance against a traveling automobile. The effect of the push of the wind

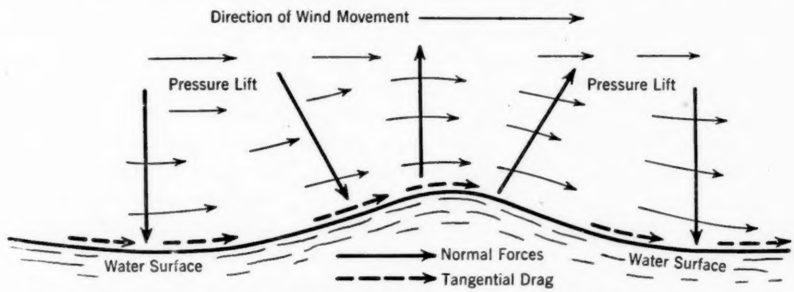


Fig. 8.—Schematic Representation of Wind System and Forces Generating Oscillatory Waves

or of the air resistance against the wave depends on the wave form. There enters, therefore, a fundamental coefficient that is related to the degree to which the wave is streamlined and which is called the "sheltering coefficient." The determination of this coefficient is necessary for the exact evaluation of energy transfer by push, that is, by normal pressures.

The pulling force of the wind is analogous to the shear of skin or pipe friction and always acts in the direction of the wind. It is the same at the wave crest and the wave trough, but the effect differs. Energy is transferred from the air to the water (the movement of the surface is speeded up) if the surface water moves in the direction of the wind; but energy is given off from the water to the air. (The movement of the surface water is slowed down) if the surface water moves against the wind. When wind and waves move in the same direction, the water particles at the crest move in the direction of the wind drag, while those in the trough move against the drag (see Figs. 3 and 8). In the absence of a mass transport velocity, the particle velocities at the crest and the trough are equal but in opposite directions, so that the effect of the pulling force of the wind at the wave crest is exactly balanced by the effect at the wave trough. However, in the presence of a mass transport velocity, the forward motion at

the crest is greater than the backward motion in the trough, and a net amount of energy is transferred to the water. No satisfactory explanation of the growth of waves can be given without assuming a transfer of energy because of the wind pulling at the water particles; and this fact is the best argument for the presence of a mass transport velocity in ocean waves.

The pulling force, or shear, of the wind over the ocean can be determined by methods used in meterology, and then the energy transfer from the air to the water by wind drag can be computed with considerable accuracy, using the theoretical values for mass transport velocity. Even when the wave velocity exceeds the wind velocity, the effect of the wind drag remains nearly the same because it depends on the difference between wind velocity and particle velocity in the water. In general, the water particles move much more slowly than the wind even when the wave form moves much faster. If the wind cannot transfer energy to the water by pulling at the water particles, no satisfactory explanation can be given of the fact that waves frequently have a higher velocity than the wind that produces them.

Energy is dissipated in the wave by viscous shear, but the viscosity of the water is so slight that these losses can be neglected in the generating process. There is no evidence that energy is dissipated by turbulent motion in the wave. Therefore the chief processes that can alter the wave height or the wave velocity in deep water are the normal pressure or push of the wind, which becomes an air resistance if the wave travels faster than the wind, and the drag or shear of the wind on the sea surface (which adds energy to the wave so long as the wind velocity exceeds the water particle velocities). The wave thus grows larger until a balance is reached between the energy added by shear and that lost by air resistance.

Knowing the rate of energy transfer from the wind and the rate at which the wave energy advances, it is possible to establish a differential equation from which the relationships between the waves and wind velocity, fetch, and duration are obtained as special solutions. The equation contains three numerical constants (including the "sheltering coefficient") that have to be determined in such a manner that nine empirical relationships are satisfied. This has been accomplished, and at the same time discrepancies between existing empirical relationships have been accounted for.

The maximum wave that can be generated is thus determined by: (1) the wind velocity; (2) the interval of time during which that wind velocity exists, or the wind duration; and (3) the distance over which the wind blows with that velocity, or the fetch.

The Jeffreys Theory.—The second hypothesis, formulated by H. Jeffreys, is similar to the first in its major concepts, except that it is assumed that energy transfer takes place by normal pressures (push) only. Therefore no net transfer of energy can occur once the wave speed equals the wind speed, and no wave can travel faster than the wind that generated the wave.

Analysis of Hypotheses.—The relatively meagre amount of observational data available to confirm the hypotheses does not permit an absolute confirmation of either; however, what is available appears to show the Sverdrup-Munk hypothesis in closer agreement with the data than the Jeffreys hypothesis.

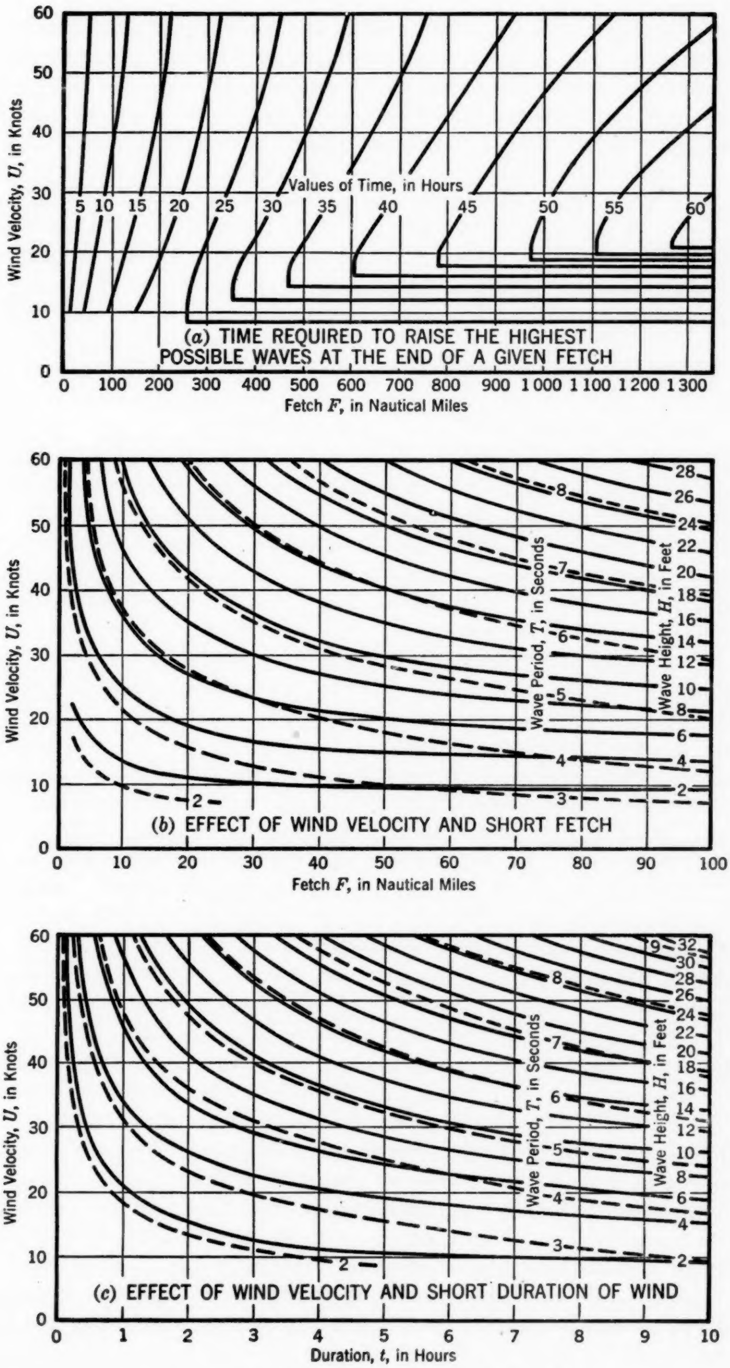


Fig. 9.—Short-Term Growth of Wind Waves

Speaking generally, the hypothesis of Messrs. Sverdrup-Munk leads to agreement of the order of $\pm 25\%$ with observed data, but there is as yet no confirmation of their concept of the details of the generation process.

It should be noted that the hypotheses discussed involve the concept of an area in which the waves are being generated and have not reached a stable form. They involve also the concurrent concept of a decay area, outside the generating area, in which the waves are of stable form but losing energy, slowly by air resistance, and very slowly by internal friction, as they travel away from the generating area. It is apparent that any single wave or series of waves in the ocean may travel through one or more generating and decay areas in its course toward a shore. In a lake, a reservoir, or another body of water of limited extent, the wave is probably always in its generating area and may not attain a stable maximum form before reaching shore. Many engineering problems of wave action involve the latter situation and justify extensive further study of waves within a generating area.

Messrs. Sverdrup and Munk have prepared useful charts for the prediction of wave heights and periods. The reliability of this data for situations of limited fetch and duration times, for instance, in the case of lakes or reservoirs, is open to question. However, tests have shown a fair agreement between observed and computed wave characteristics (see Fig. 11). The empirical formulas of T. Stevenson, D. A. Molitor, and W. P. Creager, and others do not yield results comparable in accuracy of agreement with observation to those obtained from the Sverdrup-Munk formula. Therefore, it must be concluded that they are inferior to that formula for engineering purposes. The growth of waves in terms of the wind velocity, the fetch, and the time required to generate the highest possible waves is shown in Fig. 9(*a*). This chart is used to determine the minimum length of time (duration) the wind must blow at a given velocity and over a given fetch to generate the highest possible waves that can be generated under those conditions of velocity and fetch.

Figs. 9(*b*) and 10(*a*) show the relationships between wave period or wave height, wind velocity, and fetch. Fig. 9(*b*) is the short fetch portion of Fig. 10(*a*) reproduced to a larger scale. These figures are used to determine the height and period of the wave generated.

Since either fetch or wind duration may be the controlling factor in wave generation, the relations shown in Figs. 9(*c*) and 10(*b*) between wave height or period, wind velocity, and wind duration have been prepared.

The use of these graphs can be illustrated by a simple example. Let it be supposed that a wind of 30 knots velocity blows over a fetch of 450 nautical miles for a period of 30 hr. Fig. 9(*a*) is used to determine whether fetch or duration is the limiting factor in the wave generation. The graph shows that the highest possible wave that can be generated under these conditions will be generated, since a 30-knot wind blowing over a 450-mile fetch requires just 30 hr. to generate the maximum wave. Fig. 10 is now used to determine the height and period of the generated wave. Entering Fig. 10(*a*) on the left at a wind velocity of 30 knots and moving to the intersection of this line with 450-mile fetch, a wave height of 19.5 ft is found with a corresponding wave period of 9 sec.

Suppose, however, that the wind blows for a time of only 20 hr. Fig. 9(*a*) then shows that wind duration is the control and that Fig. 10(*b*) must be used to determine the wave characteristics. Entering Fig. 10(*b*) on the left at a wind

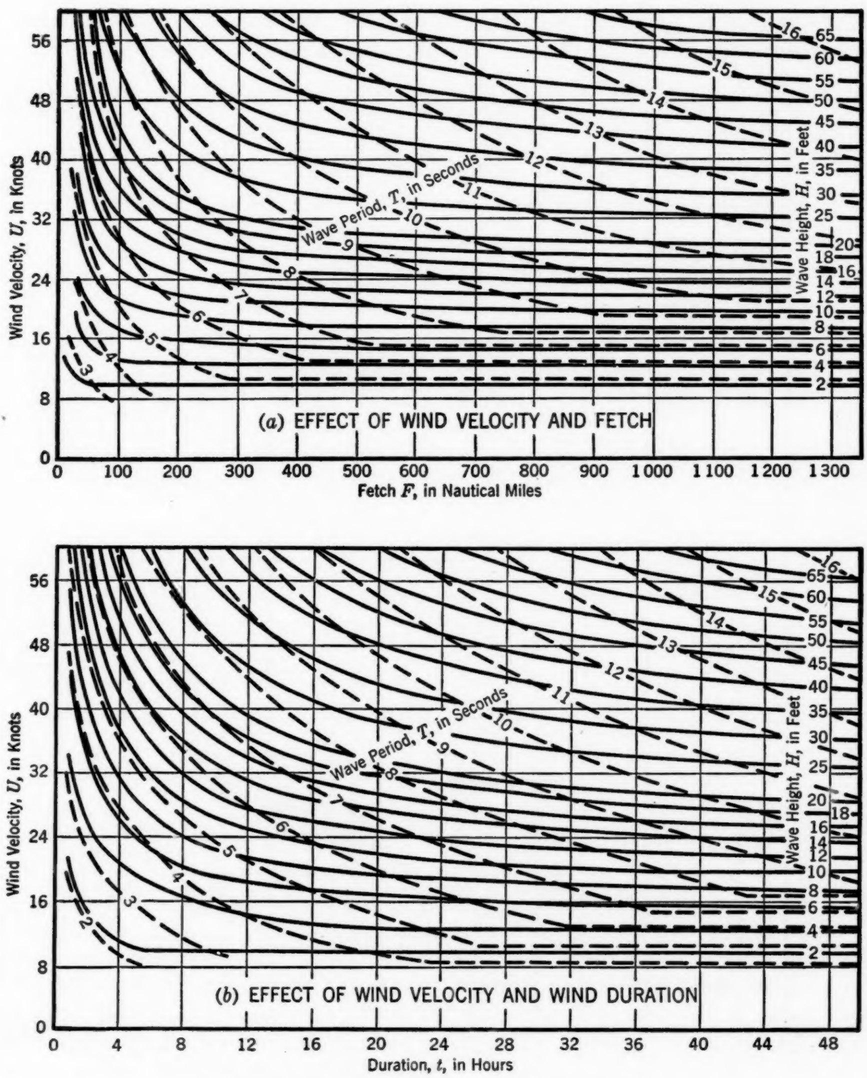


FIG. 10.—LONG-TERM GROWTH OF WIND WAVES

velocity of 30 knots and moving to the intersection with 20 hr a wave height of 16.5 ft and a wave period of 7.6 sec is found.

Fig. 11 shows a comparison between wave heights and periods as predicted by the Sverdrup-Munk method and as observed in tests at Abbotts Lagoon.

In this figure, T is the wave period; H is the wave height; F is the fetch; and u is the wind speed 8 meters above water surface. The data apply to short fetches and are presented in dimensionless form for convenience of comparison. Fig. 12 is presented primarily to show the observations of waves whose velocity exceeds that of the generating wind (values of wave age in excess of one) supporting the Sverdrup-Munk hypothesis of wave generation.

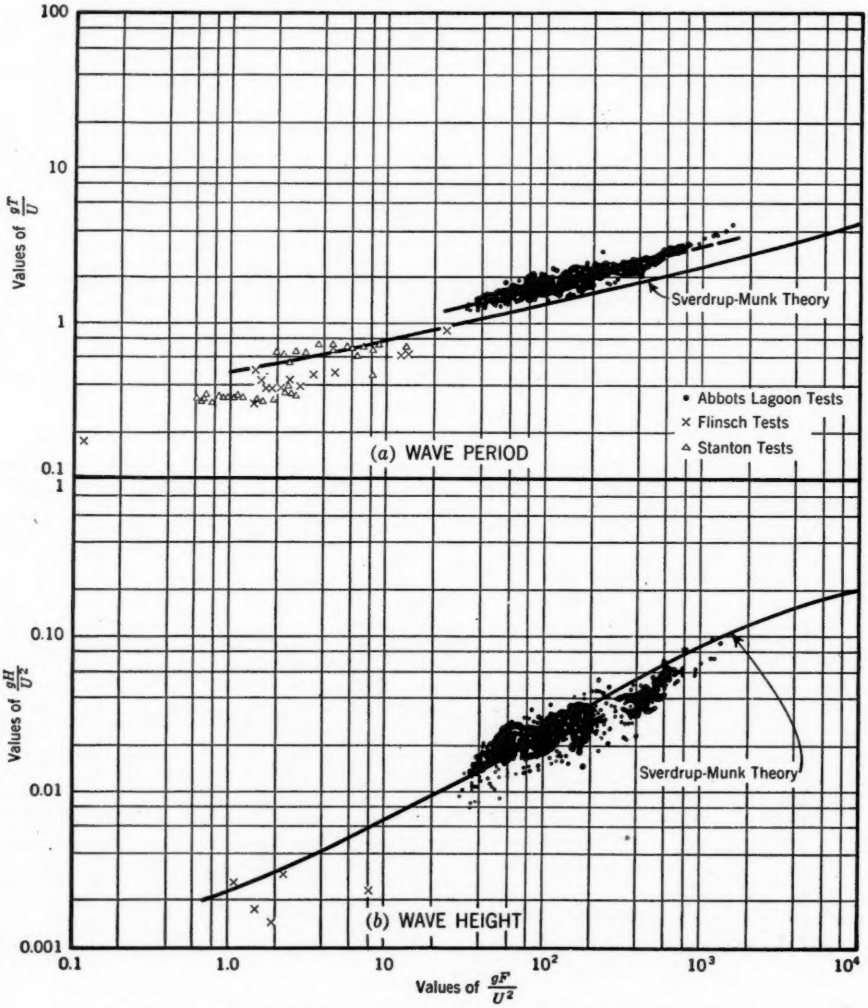


FIG. 11.—COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

Wave Refraction, Diffraction, and Reflection.—From the engineering viewpoint the problems of wave reflection, refraction, and diffraction are probably of more interest than any others, except those involving wave forces. Observation of wave behavior in nature and in the laboratory has led to the conclusion

that progressive oscillatory waves in water behave in a manner similar to light, thus leading to the use of geometrical optics and the wave theory of light to solve water wave problems. There is presumed to be an analogy between the individual water wave and the light wave, as well as between direction of travel of the water wave and light rays. If this presumption is true, then Descarte's law stating that the angle of incidence is equal to the angle of reflection is applicable to the water wave and provides a means of solving water wave refraction problems; the Snell law of refractions is applicable to refraction problems; and the laws of Huyghens and Fresnel provide methods of solving diffraction problems. In the application of these laws it must be remembered that the velocity of a water wave is a function of the water depth, and proper account

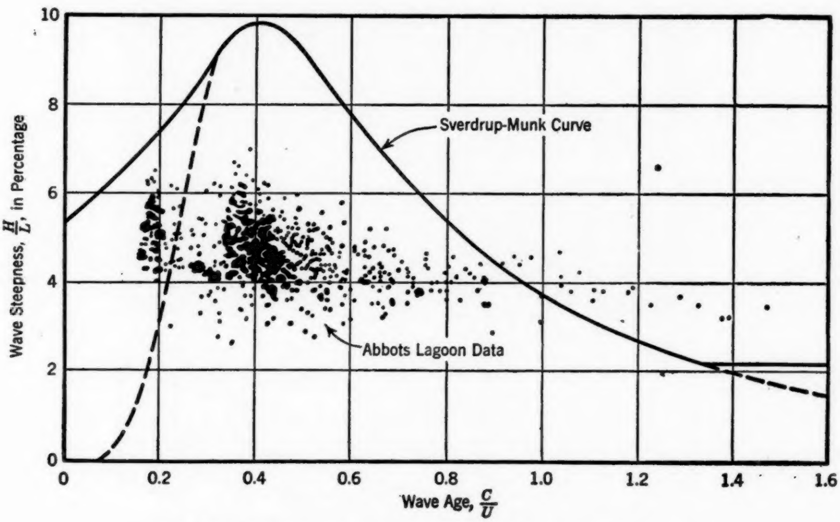


FIG. 12.—RELATIONSHIP BETWEEN WAVE STEEPNESS AND AGE

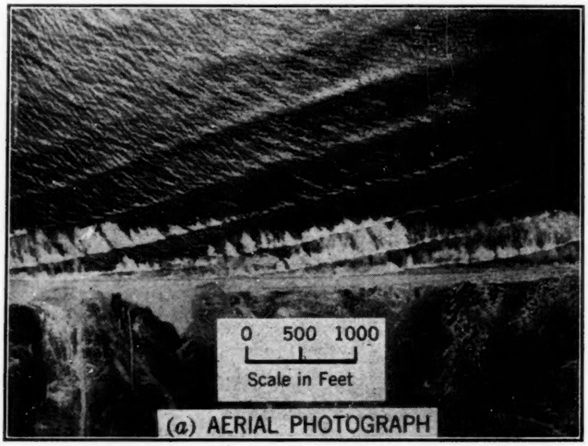
must be taken of this condition; otherwise the usual and well-known procedures of geometrical optics are applicable. Application of these laws to many situations is discussed by H. Gridel (24).

The author is not aware of any work that has been done to verify Mr. Gridel's application of geometrical optics to water wave reflection problems. However, other work on refraction and diffraction problems has confirmed Mr. Gridel's beliefs, and it can be presumed that reflection problems can be treated adequately by these methods. These problems, as well as those of refraction and diffraction, probably are most easily solved by graphical methods, although computation procedures may be applied.

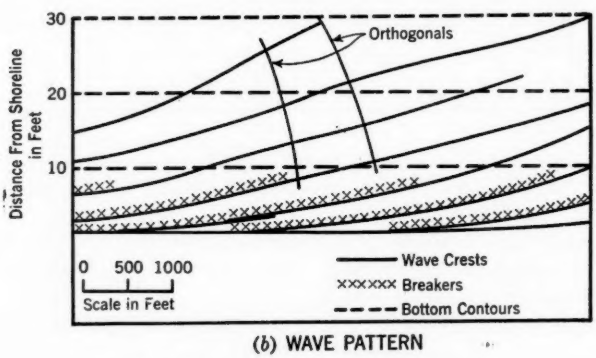
In view of the large number of possible engineering problems involving wave reflection, refraction, and diffraction and the high development of methods of geometrical optics, no discussion will be included in this paper of specific applications. The reader is referred to the "Appendix-Bibliography on Wave Theories" and to a paper by J. W. Johnson for information on the details of applicable laws and methods available for solution of such water wave problems.

It must be stated that few problems of this type have been solved for engineering purposes, but the theoretical concepts are available and merely await application.

Recapitulating, it was noted previously in discussion of the characteristics of oscillatory waves that water depth affected the height, length, and velocity of a wave whenever the depth was less than about one-half the wave length.



(a) AERIAL PHOTOGRAPH



(b) WAVE PATTERN

FIG. 13.—DETERMINATION OF WAVE PATTERN

It is believed that depth has no effect on the period of the wave; further it is assumed, and observation confirms the assumption in gross for a uniform system of waves, as in a wave channel, waves maintain their identity in running over a sloping bottom and their periods remain constant. Theory and observation are in close agreement also in showing that wave steepness increases by reason of a decrease in wave velocity (therefore wave length) and an increase in wave height. The dissipation of energy by internal and bottom friction has been assumed to be negligible for engineering purposes, and the assumption is confirmed in gross by experiment.

A typical example of wave refraction is illustrated in Fig. 13, which is an aerial photograph of swell, breakers, and surf north of Oceanside, Calif. Since the wave velocity decreases with a decrease in water depth, the inshore end of the wave travels at a slower rate than the offshore end, thus bending (or refracting) the wave and causing its direction of approach to tend more and more toward a perpendicular to the shore. It is evident that analogous sections of each wave in the series pass through the same sequence of angles and velocities

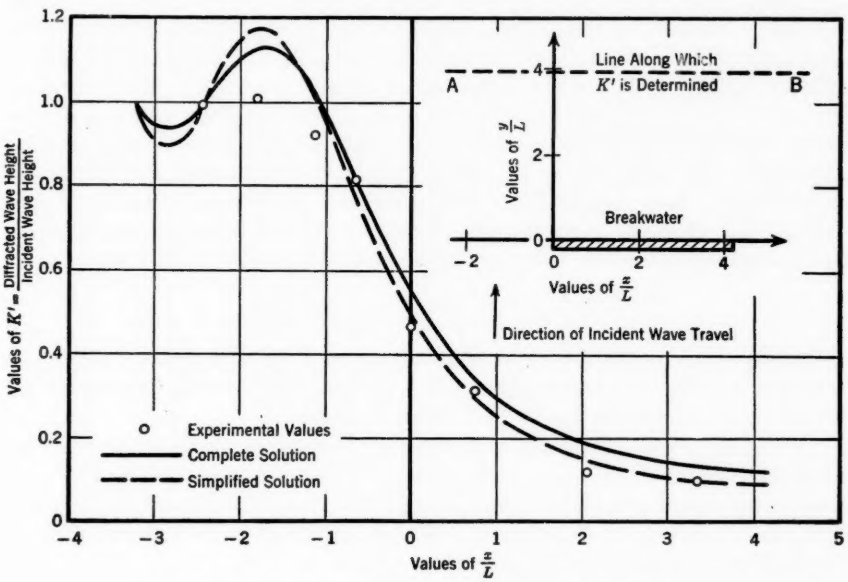


FIG. 14.—SOLUTION OF WAVE DIFFRACTION PROBLEM

as they approach the shore. Graphical solution of the problem is indicated (27), (32). The technique is highly useful in engineering problems involving oscillatory wave action, and when it is applied by competent personnel it gives quite accurate results.

In the forecast or prediction of wave action in shallow water, refraction effects are of prime importance. Application of both wave prediction and wave refraction theory to synoptic weather data permits prediction at the shore of wave direction, wave height, wave velocity, height of breakers, character of the surf, and the depth in which waves will break.

Wave diffraction so named because of its nature being analogous to the same phenomenon in light (as in the case of wave refraction) is defined as the propagation of waves into an area sheltered by an obstruction that interrupts a portion of a wave travelling beyond the obstruction. Wave diffraction theory is similar to the solution of the optical diffraction problem offered by A. Sommerfeld (34), and is described in detail by J. A. Putnam (35), R. S. Arthur, and others (36, 37). The theory assumes unnatural conditions, but agreement with observation in nature is good nonetheless. The solution of a typical case in-

volving a harbor breakwater is shown in Fig. 14, as an illustration of the application of the method.

Discussion of oscillatory wave reflection phenomena is required to be unsatisfactory, since little study has been devoted to the subject. Students of wave theory appear to be in accord generally that the analogy to light reflection is applicable in this case, as in the cases of refraction and diffraction. Mr. Gridel states that the analogy is complete and gives several illustrative examples to prove the validity of his opinion without, however, the proof of experimental evidence. There is certainly little, if any, evidence available to contradict his treatment of the problem. Studies have been made by Joseph M. Caldwell (39), Assoc. M. ASCE, of the reflection of solitary waves, and the results are believed to be applicable to oscillatory waves. The engineer concerned with such problems is referred to the bibliography of this paper for sources of detailed information, but he is warned that stock solutions of his problems probably will not be found. However, as Mr. Gridel states, he probably will find that application of the methods of optics will permit the solution of problems for which there does not appear now to be any other satisfactory method of solution.

Wave Action on Structures

Development of the theory of wave action on structures is incomplete, and consequently available knowledge is both limited and somewhat uncertain by reason of lack of verification of theoretical concepts.

Two cases of wave action on structures are recognized: In the first the waves act on a structure as unbroken surface waves with reflection of the waves being minor to an extent permitting its neglect; in the second the waves are (essentially) totally reflected by the structure and may approach as unbroken waves, or as surf (that is, during or following breaking).

Typical of the first case is the problem studied by J. R. Morison (43), Morrough P. O'Brein (M. ASCE), J. W. Johnson (M. ASCE), and S. A. Schaaf, who state that the force exerted by unbroken surface waves on a cylindrical object is made up of two components:

1. A drag force proportional to the square of the particle orbital velocities, which may be represented by a drag coefficient having substantially the same value as for steady flow;
2. A force proportional to the horizontal component of the accelerative force exerted on the mass of water displaced by the object.

Since both these forces are caused by the orbital motion in wave action, the forces are oscillatory, reversing in direction and varying in magnitude during the wave cycle. Laboratory investigations lead to the recommendation of the following expressions as sufficiently accurate for design purposes, pending the completion of more extensive development. Mr. Morison and his associates propose as the expression for the maximum force exerted at any particular depth z:

$$\left(\frac{dF}{dz}\right)_{\max} = \frac{\pi^2 \rho D H^2}{2 T^2}\left(f_d + \frac{f^2_m}{f_d}\right) \qquad (7)$$

in which ρ is the water mass density, in slugs per cu ft; D is the pile diameter, in feet; H is the wave height, in feet; T is the wave period, in seconds; $f_d =$ (1.626 ± 0.414)

$$\left[\frac{\cosh \dfrac{2\,\pi\,(d+z)}{L}}{\sinh \dfrac{2\,\pi\,d}{L}}\right]^2; \text{ and } f_m = (1.508 \pm 0.197)\,\frac{\pi\,D}{H}\left[\frac{\cosh \dfrac{2\,\pi\,(d+z)}{L}}{\sinh \dfrac{2\,\pi\,d}{L}}\right].$$

Their expression for the moment about the bottom of the pile caused by wave force is:

$$M = \frac{\rho\,D\,H^2\,L^2}{T^2}\left(-\frac{D\,\pi}{4\,H}\,C_m\,K_1 \sin\theta \pm C_d\,K_2 \cos^2\theta\right) \quad (8)$$

in which

$C_m = 1.508 \pm 0.197$; $C_D = 1.626 \pm 0.414$; $\sin\theta = f_m/2\,f_d$; $\cos^2\theta$

$$= \frac{4\,f^2_d - f_m}{4\,f^2_d};\ K_1 = \frac{1 + \dfrac{2\,\pi\,d}{L}\sinh\dfrac{2\,\pi\,d}{L} - \cosh\dfrac{2\,\pi\,d}{L}}{2\sinh\dfrac{2\,\pi\,d}{L}};\ \text{and } K_2$$

$$= \frac{1 + \frac{1}{2}\left(\dfrac{4\,\pi\,d}{L}\right)^2 + \dfrac{4\,\pi\,d}{L}\sinh\dfrac{4\,\pi\,d}{L} - \cosh\dfrac{4\,\pi\,d}{L}}{64\left(\sinh\dfrac{2\,\pi\,d}{L}\right)^2}.$$

Mr. Munk has treated the same problem on the theoretical concept that only the drag forces need be considered, arriving at the maximum force expression

$$F = C_D\,D\,H^2\,K_f \quad (9)$$

in which C_D depends upon the object shape and Reynolds number ($C_D = 0.33$ for a cylindrical pile), and $K_f = \dfrac{g}{8}\left[1 + \dfrac{\dfrac{4\,\pi\,d}{L}}{\sinh\dfrac{4\,\pi\,d}{L}}\right]$.

His expression for the moment about the bottom of the pile caused by wave force is

$$M = C_D\,D\,H^2\,d\,K_m \quad (10)$$

in which

$$K_m = \frac{g}{8}\left[1 + \frac{\dfrac{2\,\pi\,d}{L}}{\sinh\dfrac{4\,\pi\,d}{L}} - \frac{\tanh\dfrac{2\,\pi\,d}{L}}{\dfrac{4\,\pi\,d}{L}}\right].$$

Neither of the theories noted has received confirmation from actual observations of forces or moments; however the Morison theory shows close agreement with laboratory observations.

The second case of wave action on structures, involving reflection of the waves in an unbroken or broken state, has received considerable attention by foreign students. Their interest was aroused by a number of failures of vertical face breakwaters under the stress of wave action, and their theories are concerned primarily with such situations. The most generally accepted theory is that developed by G. Sainflou in 1928 from earlier studies by M. Benezit in 1923. In the Sainflou theory it is considered that essentially total reflection of the wave occurs, giving rise to a clapotis (standing wave), and the resulting pressures and forces are computed from standing wave theory.

The expression for maximum pressure at any depth d_i is

$$P = d + H\left[\frac{\cosh \dfrac{\pi\,(d - d_i)}{L}}{\cosh \dfrac{\pi\, d}{L}} - \frac{\sinh \dfrac{\pi\,(d - d_i)}{L}}{\sinh \dfrac{\pi\, d}{L}}\right] \qquad (11)$$

in which d is the water depth, in feet; H is the wave height, in feet; and d_i is the depth to any point, in feet.

The maximum moment referred to the base of the structure is

$$M = \frac{\left(d + \dfrac{\pi\, H^2}{2\,L} \coth \dfrac{\pi\, d}{L} + H\right)^2}{6}\;\frac{\left(d + \dfrac{H}{\cosh \pi\, d/L}\right)}{\dfrac{\pi\, d}{L}} - \frac{d^3}{6} \qquad (12)$$

in which d is the still water depth, in feet; H is the wave height, in feet; and L is the wave length, in feet.

It should be noted that for complete design analysis both maximum and minimum pressures should be computed, as well as the distribution of pressures (45).

The problem of the size of stone and side slopes to be used in the construction of breakwaters, sea walls, or revetments to resist wave action has been treated as a special problem by R. Iribarren (42). The Iribarren theory presumes that disintegration or failure of the structure occurs by reason of displacement of the individual stones of the structure. This displacement is caused by resultant forces on the faces of the stone, due to wave action, exceeding the weight and keying forces associated with the stone size, shape, and position. The theory does not lead to a rigorous solution; however, Mr. Iribarren has derived the following empirical relationship that is being used increasingly for design purposes:

$$P = \frac{K\, A^3\, \rho}{(\cos \alpha - \sin \alpha)^3\, (\rho - 1)^3} \qquad (13)$$

in which P is the weight of stone, in kilograms; A is the height of wave acting on the structure, in meters; ρ is the density of stone, in tons per cubic meter;

α is the side slope of dike, angle with horizontal; and K is the coefficient, having a value of 15 for natural rock shapes and 19 for artificial blocks.

Although several structures in the United States are understood to have been designed according to the Iribarren criterion, there is not sufficient confirmation of the formula available to obviate the need for further study and test. The author is aware that Spanish, Portugese, and French engineers also are using the Iribarren formula, and it appears that it enjoys the confidence of many engineers engaged in the design of maritime structures.

Transportation of Beach and Bottom Material

A phenomenon of engineering importance involving wave action is that of the movement of beach and bottom material by waves. No complete and adequate theory of transportation by waves has been developed; however, there has been much research on the problem. Available concepts of the transportation phenomena are included in the following description of what has been observed or is believed to occur.

As noted previously, there is a motion of water particles associated with wave action, the motion being a maximum at the surface and decreasing to zero at a depth approximately half the wave length. As a wave progresses into shallow water, this water motion approaches the bottom; beginning at a depth of about $L/2$ there exists a velocity field contiguous to the bottom. As the wave moves shoreward, the velocities at the bottom increase until they are sufficiently high to induce creep, saltation, and suspension of the bottom material. However the velocities are not uniform or steady but periodic and reversing. Experiments at the Beach Erosion Board and elsewhere have confirmed that movement of bottom material in this region does occur and that it is frequently accompanied by sand ripple formation on the bottom. Farther shoreward the wave breaks, rushes up the beach, pauses for an instant, and then flows back down the beach slope. Experiments have shown highly turbulent flow conditions and a very high percentage of suspended material to exist in the breaker region. The Beach Erosion Board is engaged in making high speed photographic studies of this region in an effort to define its behavior.

The material thus stirred up and placed in suspension is carried by the uprush of the wave onto and along the beach, and to some extent seaward. Laboratory tests have shown the transportation to be related in some fashion to the wave steepness, net movement to the beach being associated with low steepness, net movement seaward accompanying waves of high steepness. It is believed that as much as 80% of the material moved by wave action is moved in the area from the point of breaking waves to the limit of uprush of the waves on the beach, as shown in Fig. 15. In this figure, sand movement is given in zones parallel to the shore.

Hydraulic Models of Waves

As stated previously, perhaps the most obvious characteristic of wave motion in nature is its apparent confusion. This inherent variability makes study of waves in nature an expensive and difficult task, with the result that most developments in wave study have been dependent on model experimentation.

It is almost certain that this situation will continue. It is then pertinent to examine the possibilities of hydraulic wave model experimentation and the reliability of such results.

As a general requirement it can be stated that an hydraulic model of a wave situation requires complete geometric similitude; that is, no model distortion is permissible. The requirement arises from the fact that the characteristics of the wave, other than period, are functions of the water depth, and can be re-

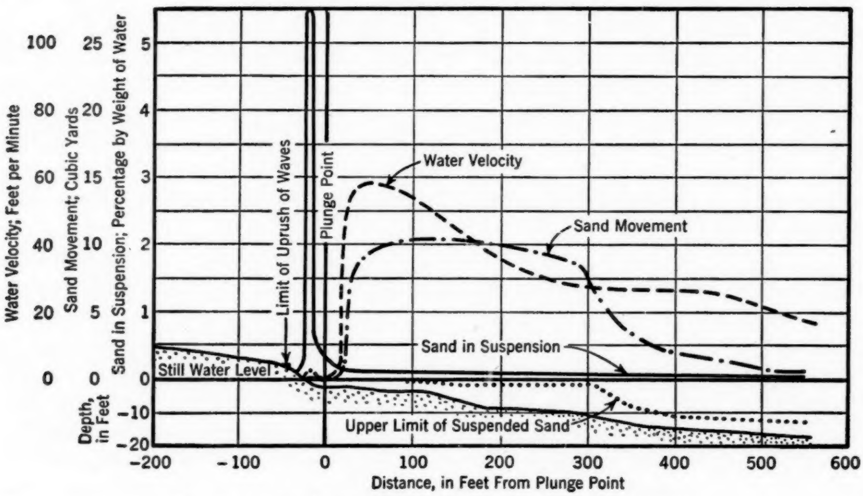


FIG. 15.—SAND MOVEMENT AT LONG BRANCH, NEW JERSEY

laxed only for wave problems that do not involve waves in shallow water, that is depths less than $L/2$.

Mr. Johnson (66) has summarized the various instances in which wave theories have been verified experimentally and in which wave models at several scales have permitted an evaluation of the scale effect in the application of data from hydraulic wave models. He states that, in general, basic wave theory has been confirmed independently both in nature and in model, this confirmation in itself furnishing the strongest proof that hydraulic wave models can be made. Experiments on models of various scales and in nature have shown similar characteristics of wave action in the surf zone insofar as these characteristics can be observed, of longshore currents, and of beach profiles. Model tests at various scales have shown that breakwater stability can be studied by model experimentation based on the Froude law.

It is the author's belief that model experimentation is an admissible and powerful tool for the study of wave action and many related phenomena, but that an understanding of the physics of the situation to be studied in model is a necessary requisite to its successful use. The most certain procedure available is the application of classical model theory to the equations defining the dynamics of the wave situation. When such equations do not exist or when departures from established laws are required, confirmation of the model results should be considered as a necessary part of the study.

Conclusion

The hydraulic engineer who finds himself concerned with oscillatory wave problems should consider this paper as but a very brief introduction to a body of knowledge and literature that has grown at an astonishing rate since 1940. It will be apparent to him relatively soon in his studies that, although much progress has been made, there remain multitudes of unsolved problems of both a theoretical and a practical nature. The purpose of this paper will be served if it awakens a consciousness in the practicing engineer that the knowledge and literature exist. Most of his problems will require further development of knowledge and methods of applying presently available knowledge for their solution. The field is rich in problems, but a high degree of understanding of the mechanics involved and an appreciation of the importance of sound theoretical concepts must be brought to their solution.

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